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Form factors of heavy-to-light B decays at large recoil

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Abstract

General relations between the form factors of B decays to light mesons are derived using the heavy quark and large recoil expansion. On their basis the complete account of contributions of second order in the ratio of the light meson mass to the large recoil energy is performed. Both ground and excited final meson states are considered. It is shown that most of the known form factor relations remain valid after the inclusion of quadratic mass corrections. The validity of some of such relations requires additional equalities for the helicity amplitudes. It is found that all these relations and equalities are fulfilled in the relativistic quark model based on the quasipotential approach in quantum field theory. The contribution of $1/m_b$ corrections to the branching fraction of the rare radiative B decay is discussed.

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I. INTRODUCTION

The investigation of exclusive weak B decays to mesons composed of the light u, d, s quarks (heavy-to-light decays) represents an important problem in particle physics. It provides means to measure such fundamental parameters as the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{ub}, V_{td} and V_{ts}, to predict CP-violating asymmetries, to test the Standard Model both at large and short distances and to look for possible manifestations of new physics. In contrast with B decays to D mesons, where both initial and final mesons contain the heavy b and c quarks (heavy-to-heavy decays) and thus well developed methods of heavy quark effective theory (HQET) [1] can be applied, the heavy-to-light decays are less studied theoretically. HQET and heavy quark spin-symmetry do not reduce the number of independent form factors for each decay mode in the latter case, but yield only relations between form factors of different heavy-to-light modes, e. g. semileptonic and rare radiative B decays as in the Isgur-Wise relations [2].

In heavy-to-light decays the final meson usually carries a large recoil momentum (energy) of order of half a B meson mass. The inverse of this large quantity can serve as an expansion parameter of corresponding large energy effective theory (LEET). It was used originally by Dugan and Grinstein [3] in studying nonleptonic two-body B decays to a charmed and a light meson (so-called energetic decays). Further development was given in Ref. [4].

We used a similar approach within the relativistic quark model while considering exclusive rare radiative B decays to K^* and γ [5]. For the real photon ($q^2 = 0$) the K^* recoil momentum Δ has a fixed value $(M^2 - m_{K^*}^2)/(2M)$ and its energy is equal to $(M^2 + m_{K^*}^2)/(2M)$. Since the B meson mass $M \gg m_{K^*}$ and M is of order of the b quark mass m_b, both the recoil momentum and energy are of order of $m_b/2$. Thus we can make an expansion in $1/\Delta \sim 2/m_b$ for the final state and the usual $1/m_b$ heavy quark expansion for the initial one. These expansions significantly simplify calculations of the matrix element. Later we applied the same approach to the description of exclusive semileptonic heavy-to-light B decays [6]. In this case the value of the light meson recoil momentum is not fixed, but still is of order of $m_b/2$ almost in the hole kinematical range. The Isgur-Wise relation originally derived for the heavy-to-light form factors at maximum $q^2_{\rm max} = (M - m_V)^2$ is satisfied in our model for small $q^2 \ll M^2, m_V^2$ (near the point of maximum recoil, m_V is the mass of a final vector meson). A close approach was used by Stech [7] and Soares [8] for describing heavy-to-light transitions.

Recently it has been shown [9] that in the heavy quark and large recoil limit new symmetries emerge and impose new relations on the form factors (thus reducing the number of independent ones) of heavy-to-light B decays. The interaction with collinear gluons preserves these relations, establishing them in the large energy limit of QCD [10]. On the other hand these new symmetries are broken by radiative corrections [11].

In this paper we extend the analysis of Ref. [9] to include systematically the contributions quadratic in the final to initial meson mass ratio and of order q^2/M^2. Their inclusion is especially important when the final meson is in the radially or orbitally excited state and the above contributions become quite appreciable. To achieve this goal the general expressions for the heavy-to-light form factors accounting for nonzero final meson mass are derived. They allow to obtain many important relations between form factors. In particular, the inclusion of these contributions is necessary for the exact fulfilment of the Isgur-Wise relations [2] and some of the relations given by Soares [8]. Additional constraints which are specific for the constituent quark model lead to the fulfilment of the rest of the relations found by Soares [12].

The paper is organized as follows. In Sec. II we consider the effective theory for the description of heavy-to-light decays at large recoil of the final meson. Special emphasis is made on the inclusion of the corrections of second order in the ratio of the final meson mass to its recoil energy. Such corrections are especially important for B decays to excited light mesons since their mass is not small enough. The general formula for calculating weak decay matrix elements in the effective theory is given. In Secs. III and IV the effective theory is applied for deriving various symmetry relations between the form factors of B decays to ground and radially or orbitally excited meson states. A significant reduction in the number of independent decay form factors in the heavy quark and large recoil limit is observed. The obtained form factor relations are compared to known ones. The fulfilment of form factor relations in the relativistic quark model is tested in Sec. V. Then, some phenomenological applications are discussed. Finally, we give our conclusions in Sec. VI.

II. THE LARGE RECOIL EFFECTIVE THEORY

We consider the kinematical region where the final meson energy $E_F = (M^2 + m^2 - q^2)/(2M)$ is of order $M/2$, $m^2 \ll M^2$ and the four-momentum transfer squared $q^2 \ll M^2$. This means that the final light meson bears a large recoil three-momentum $\Delta \sim M/2$. We assume that the main part of this momentum is carried by an active light quark in the final meson and that its interaction with the spectator light quark is soft, hence we neglect all hard gluon contributions. We also neglect the Sudakov suppression of the soft part of the form factors [9,11]. Since the B meson mass $M \sim m_b$, we use the $1/M$ expansion retaining all terms of order m^2/M^2 and neglecting the ones of order $\Lambda_{\rm QCD}/M$ and higher.[1] To this end we introduce the following kinematical notations: the four-momentum of the heavy B meson with mass M and velocity v

$$p_B = Mv, \tag{1}$$

the heavy quark momentum

$$p_Q = m_Q v + k, \tag{2}$$

where m_Q is the heavy quark mass and k is a small residual momentum ($|k| \sim \Lambda_{\rm QCD} \ll m_Q$).

The energetic final light meson carries a momentum

$$p_F = En + \frac{m^2}{4E}\eta, \tag{3}$$

where we have introduced two light-like vectors n_μ and $\eta_\mu = 2v_\mu - n_\mu$ ($n^2 = 0$ and $\eta^2 = 0$) satisfying the relations $n \cdot v = 1$ and $n \cdot \eta = 2$. In the rest frame of the initial B meson, where $v = (1, 0, 0, 0)$, choosing the momentum of the final light meson in z direction, we get $n = (1, 0, 0, 1)$ and $\eta = (1, 0, 0, -1)$. It is easy to check that $p_F^2 = m^2$, where m is the mass of the final meson. The important difference with the case when the final meson mass is being neglected consists in that E is not the on-shell energy of the final meson. Indeed, the on-shell energy E_F and the recoil momentum $\mathbf{\Delta}$, which form the final meson four-momentum $p_F = (E_F, \mathbf{\Delta})$, are related to E by

$$E_F = \frac{M^2 + m^2 - q^2}{2M} = E\left(1 + \frac{m^2}{4E^2}\right), \tag{4}$$

$$|\mathbf{\Delta}| \equiv \Delta = \sqrt{E_F^2 - m^2} = E\left(1 - \frac{m^2}{4E^2}\right), \tag{5}$$

$$2E = E_F + \Delta, \qquad \frac{m^2}{2E} = E_F - \Delta, \qquad q = p_B - p_F.$$

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[1]The m^2/M^2 corrections are especially important for decays to excited light mesons, since their mass m is in the range $1.2 - 1.5$ GeV, i. e. about the charmed quark mass.

Under our assumptions it follows from these formulas that $E_F \sim \Delta \sim E \sim M/2$. The active light quark momentum in the final meson can be represented as

$$p_q = En + \frac{m^2}{4E}\eta + k' \equiv \Delta n + \frac{m^2}{2E}v + k', \tag{6}$$

where the residual momentum k' is small, if we neglect hard gluon exchanges, and satisfies $|k'| \sim \Lambda_{\rm QCD} \ll E$ and $|k'| \ll m$.

The two component light quark fields $q_\pm(x)$ can be defined from the full QCD fields $q(x)$ by

$$q_\pm(x) = e^{i(\Delta n\cdot x + \frac{m^2}{2E}v\cdot x)} P_\pm q(x), \tag{7}$$

where $P_\pm$ are the projectors

$$P_+ = \frac{\not n\ \not\eta}{4} = \frac{\not n\ \not v}{2}, \qquad P_- = \frac{\not\eta\ \not n}{4} = \frac{\not v\ \not n}{2}. \tag{8}$$

Substituting expressions (7) in the QCD Lagrangian $\mathcal{L}_{\rm QCD} = \bar q(i\not D - m_q)q$, one obtains

$$\begin{aligned}\mathcal{L}_{\rm QCD} = {} & \bar q_+(x)\ \not v \left(in\cdot D + \frac{m^2}{2E}\right) q_+(x) + \bar q_-(x) i \not D q_+(x) + \bar q_+(x) i \not D q_-(x) \\ & + \bar q_-(x)\ \not v (2E + 2iv\cdot D - in\cdot D) q_-(x),\end{aligned} \tag{9}$$

where the covariant derivative $D^\mu = \partial^\mu - ig_s A^\mu$. The variation of this Lagrangian over δq_- gives the equation of motion

$$\frac{\delta\mathcal{L}_{\rm QCD}}{\delta q_-} = i\not D q_+(x) + \not v (2E + 2iv\cdot D - in\cdot D) q_-(x) = 0, \tag{10}$$

which allows to express formally $q_-(x)$ in terms of $q_+(x)$

$$q_-(x) = -\frac{i \not v \not D}{2E + 2iv\cdot D - in\cdot D}\, q_+(x). \tag{11}$$

Thus $q_-(x)$ is suppressed by $\Lambda_{\rm QCD}/E$ with respect to $q_+(x)$. Expanding the Lagrangian $\mathcal{L}_{\rm QCD}$ (9) in inverse powers of $1/E$ up to leading order in $\Lambda_{\rm QCD}/E$ and keeping terms of order m^2/E^2, we get the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \bar q_n(x)\ \not v \left(in\cdot D + \frac{m^2}{2E}\right) q_n(x), \tag{12}$$

where $q_n(x) \equiv q_+(x)$. This effective Lagrangian differs from the $\mathcal{L}_{\rm LEET}$ one of Ref. [9] by the second term, which represents the correction accounting for the final meson mass. However, this correction does not violate the symmetry of the leading order Lagrangian, since it has the Dirac structure similar to the leading contribution. Thus the symmetry relations are not spoiled by this term.[2] For the complete determination of the m^2/E^2 corrections it is necessary to use the exact expression (3) for the final meson momentum and relations (4), (5) between E, on-shell energy E_F and recoil momentum Δ of the final meson. Taking this into account we obtain for the sum and difference of four-momenta of the initial and final meson

$$\begin{aligned} p_B + p_F &= M\left(1+\frac{m^2}{2ME}\right)v + \Delta n, \\ q \equiv p_B - p_F &= M\left(1-\frac{m^2}{2ME}\right)v - \Delta n. \end{aligned} \tag{13}$$

The transversality conditions for the polarization vector ϵ_μ of the (axial) vector mesons and tensor $\epsilon_{\alpha\beta}$ of the tensor mesons $\epsilon^*\cdot p_F = 0$ and $\epsilon^*_{\alpha\beta}p_F^\beta = 0$ imply that

$$\begin{aligned} \epsilon^*\cdot n &= -\frac{m^2}{2E\Delta}\epsilon^*\cdot v, \\ \epsilon^*_{\alpha\beta}n^\beta &= -\frac{m^2}{2E\Delta}\epsilon^*_{\alpha\beta}v^\beta. \end{aligned} \tag{14}$$

The symmetry relations between soft form factors of weak B decays to excited light mesons in the large recoil limit can be obtained using methods of Ref. [11]. The matrix element of the quark current between the initial B meson state in the infinitely heavy quark limit and the final state of the excited light meson at large recoil is given by

$$\langle F(p_F)|\bar q_n \Gamma h_v|B(p)\rangle = \mathrm{tr}[A_F(E_F)\bar{\mathcal{M}}_F\Gamma\mathcal{M}_B], \tag{15}$$

where $F = P, V, S, A, T$ for pseudoscalar, vector, scalar, axial vector and tensor, respectively. The matrices $\mathcal{M}_F$ and $\mathcal{M}_B$ have the form

$$\bar{\mathcal{M}}_F = \left\{\begin{array}{c} -\gamma_5 \\ \not\epsilon^* \\ 1 \\ -\not\epsilon^*\gamma_5 \\ \epsilon^*_{\mu\nu}\gamma^\mu v^\nu \end{array}\right\}\frac{\not v\,\not n}{2} \qquad \begin{array}{c} F=P \\ F=V \\ F=S \\ F=A \\ F=T \end{array}, \qquad \mathcal{M}_B = -\frac{1+\not v}{2}\gamma_5, \tag{16}$$

where ϵ_μ and $\epsilon_{\mu\nu}$ are the polarization vector and tensor of the (axial) vector and tensor mesons, respectively. The functions $A_F(E_F)$ are independent of the Dirac structure Γ of the current, since there are no dynamical (i. e. contracted with the covariant derivative D) Dirac matrices in the effective Lagrangian (12), and parametrize the long-distance dynamics. The most general form of $A_F(E_F)$ is [11]

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[2]The arising invariant functions in this case depend not only on the recoil energy but on the final meson mass as well. However, this dependence does not break the constraints on the form factors imposed by LEET, since the decay matrix elements for each final meson state are described by their own set of invariant functions which is the same as in LEET.

$$A_F(E_F) = a_{1F}(E_F) + a_{2F}(E_F)\, \not{v} + a_{3F}(E_F)\, \not{n} + a_{4F}(E_F)\, \not{n}\, \not{v}. \tag{17}$$

However, the presence of projection operators imply that not all the functions $a_{iF}(E_F)$ are independent. As the result, the following parametrization of $A_F(E_F)$ can be obtained

$$A_{P,S}(E_F) = 2E_F\zeta_{P,S}(E_F) \tag{18}$$

$$A_{V,A,T}(E_F) = E_F\, \not{n} \left[\zeta^{\perp}_{V,A,T}(E_F) - \frac{\not{v}}{2}\frac{m}{E_F}\zeta^{\parallel}_{V,A,T}(E_F)\right]. \tag{19}$$

Our definition of functions $\zeta_P(E_F)$, $\zeta_V^{\perp}(E_F)$ and $\zeta_V^{\parallel}(E_F)$ coincides with that of Ref. [9], but differs from [11] by an extra factor m/E_F in front of $\xi^{\parallel}_{V,A,T}(E_F)$.

III. B DECAYS TO GROUND STATE AND RADIALLY EXCITED LIGHT MESONS

First we consider the heavy-to-light B decays to ground state and radially excited pseudoscalar and vector light mesons.

A. B decays to pseudoscalar light mesons

The matrix elements of B decays to pseudoscalar mesons can be parametrized by three invariant form factors:

$$\langle P(p_F)|\bar q\gamma^\mu b|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_F^\mu - \frac{M^2-m_P^2}{q^2}\, q^\mu\right] + f_0(q^2)\frac{M^2-m_P^2}{q^2}\, q^\mu, \tag{20}$$

$$\langle P(p_F)|\bar q\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = \frac{if_T(q^2)}{M+m_P}[q^2(p_B^\mu+p_F^\mu) - (M^2-m_P^2)q^\mu], \tag{21}$$

where $f_+(0)=f_0(0)$; M is the B meson mass and m_P is the pseudoscalar meson mass.

We obtain the symmetry relations for the corresponding form factors by evaluating the trace in (15) with the account of (18), (13), (14) and find the following relations

$$\langle P(p_F)|\bar q\gamma^\mu b|B(p_B)\rangle = 2E_F\zeta_P(E_F)n^\mu, \tag{22}$$

$$\langle P(p_F)|\bar q\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = 2iE_F\zeta_P(E_F)\left[(M-E_F)n^\mu - M\left(1-\frac{m_P^2}{2EM}\right)v^\mu\right]. \tag{23}$$

The factor in front of v^μ in (23) can be expressed in terms of observable variables E_F and Δ with the help of Eqs. (4) and (5)

$$\frac{m_P^2}{2EM} = \frac{E_F-\Delta}{M}. \tag{24}$$

Thus all form factors for the B decay to a pseudoscalar meson in the heavy quark and large recoil limit can be parametrized by one function $\zeta_P(E_F)$. Comparing Eqs. (20)–(23) we find the following expressions for the form factors in terms of this invariant function

$$f_+(q^2) = \left(1 - \frac{m_P^2}{2EM}\right)\frac{E_F}{\Delta}\zeta_P(E_F),$$
$$f_0(q^2) = \frac{2E_F M}{M^2 - m_P^2}\left(1 - \frac{m_P^2}{2EM}\right)\zeta_P(E_F),$$
$$f_T(q^2) = \frac{M + m_P}{M}\frac{E_F}{\Delta}\zeta_P(E_F). \tag{25}$$

Taking into account that our consideration is valid up to second order in m_P/E and all other corrections were neglected, we omit terms of fourth order in m_P/E_F and terms of order $q^2 m_P^2/E_F^4$ in Eq. (25) and finally get

$$\Delta \cong \frac{M^2 - m_p^2 - q^2}{2M}, \qquad \frac{m_P^2}{2EM} \cong \frac{m_P^2}{M^2}, \qquad \frac{E_F}{\Delta} \cong 1 + \frac{2m_P^2}{M^2}, \tag{26}$$

$$f_+(q^2) = \left(1 + \frac{m_P^2}{M^2}\right)\zeta_P(E_F),$$
$$f_0(q^2) = \frac{2E_F}{M}\zeta_P(E_F),$$
$$f_T(q^2) = \frac{M + m_P}{M}\left(1 + \frac{2m_P^2}{M^2}\right)\zeta_P(E_F). \tag{27}$$

As a result, the following relations between the form factors of B decays to scalar light mesons in the heavy quark and large recoil limit are obtained

$$f_+(q^2) = \frac{M}{2E_F}\left(1 + \frac{m_P^2}{M^2}\right) f_0(q^2) = \frac{M}{M + m_P}\left(1 - \frac{m_P^2}{M^2}\right) f_T(q^2)$$
$$= \left(1 + \frac{m_P^2}{M^2}\right)\zeta_P(E_F). \tag{28}$$

B. B decays to vector light mesons

The matrix elements of weak current for B decays to vector mesons are parametrized by seven form factors

$$\langle V(p_F)|\bar{q}\gamma^\mu b|B(p_B)\rangle = \frac{2iV(q^2)}{M + m_V}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_{B\rho}p_{F\sigma}, \tag{29}$$

$$\langle V(p_F)|\bar{q}\gamma^\mu\gamma_5 b|B(p_B)\rangle = 2m_V A_0(q^2)\frac{\epsilon^*\cdot q}{q^2}\ q^\mu + (M + m_V)A_1(q^2)\left(\epsilon^{*\mu} - \frac{\epsilon^*\cdot q}{q^2}\ q^\mu\right)$$
$$-A_2(q^2)\frac{\epsilon^*\cdot q}{M + m_V}\left[p_B^\mu + p_F^\mu - \frac{M^2 - m_V^2}{q^2}\ q^\mu\right], \tag{30}$$

$$\langle V(p_F)|\bar{q}i\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = 2T_1(q^2)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_{F\rho}p_{B\sigma}, \tag{31}$$

$$\langle V(p_F)|\bar{q}i\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle = T_2(q^2)[(M^2 - m_V^2)\epsilon^{*\mu} - (\epsilon^*\cdot q)(p_B^\mu + p_F^\mu)]$$

$$+T_3(q^2)(\epsilon^*\cdot q)\left[q^\mu - \frac{q^2}{M^2-m_V^2}(p_B^\mu + p_F^\mu)\right], \tag{32}$$

where $2m_V A_0(0) = (M+m_V)A_1(0) - (M-m_V)A_2(0)$, $T_1(0) = T_2(0)$; m_V and ϵ_μ are the mass and polarization vector of the final vector meson. Calculating corresponding traces in (15) and accounting for (19), (13), (14), we get the heavy quark and large recoil symmetry relations

$$\langle V(p_F)|\bar{q}\gamma^\mu b|B(p_B)\rangle = 2iE_F\zeta_V^\perp(E_F)\epsilon^{\mu\nu\rho\sigma}\epsilon_\nu^* v_\rho n_\sigma, \tag{33}$$

$$\begin{aligned}\langle V(p_F)|\bar{q}\gamma^\mu\gamma_5 b|B(p_B)\rangle = 2E_F\Bigg\{&\zeta_V^\perp(E_F)\left[\epsilon^{*\mu} - \epsilon^*\cdot v\left(\frac{E_F}{\Delta}n^\mu - \frac{m_V^2}{2E\Delta}v^\mu\right)\right]\\ &+\frac{E}{\Delta}\frac{m_V}{E_F}\zeta_V^\parallel(E_F)\epsilon^*\cdot v n^\mu\Bigg\},\end{aligned} \tag{34}$$

$$\langle V(p_F)|\bar{q}i\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = 2iE_F M\zeta_V^\perp(E_F)\left(1-\frac{m_V^2}{2EM}\right)\epsilon^{\mu\nu\rho\sigma}\epsilon_\nu^* n_\rho v_\sigma, \tag{35}$$

$$\begin{aligned}\langle V(p_F)|\bar{q}i\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle = 2E_F\Bigg\{&M\zeta_V^\perp(E_F)\left(1-\frac{m_V^2}{2EM}\right)\Bigg[\epsilon^{*\mu} - \epsilon^*\cdot v\Bigg(\frac{E_F}{\Delta}n^\mu\\ &-\frac{m_V^2}{2E\Delta}v^\mu\Bigg)\Bigg] + \frac{E}{\Delta}\frac{m_V}{E_F}\zeta_V^\parallel(E_F)\epsilon^*\cdot v\Bigg[(M-E_F)n^\mu\\ &-M\left(1-\frac{m_V^2}{2EM}\right)v^\mu\Bigg]\Bigg\},\end{aligned} \tag{36}$$

All form factors for B decays to vector mesons in the heavy quark and large recoil limit can be expressed through two invariant functions $\zeta_V^\perp(E_F)$ and $\zeta_V^\parallel(E_F)$. Comparing the invariant decompositions (29)–(32) with the symmetry relations (33)–(36), we get the following expressions

$$\begin{aligned}
V(q^2) &= \frac{M+m_V}{M}\frac{E_F}{\Delta}\zeta_V^\perp(E_F),\\
A_1(q^2) &= \frac{2E_F}{M+m_V}\zeta_V^\perp(E_F),\\
A_2(q^2) &= \frac{M+m_V}{M}\frac{E_F^2}{\Delta^2}\left[\left(1-\frac{m_V^2}{E_F M}\right)\zeta_V^\perp(E_F) - \left(1-\frac{m_V^2}{2EM}\right)\frac{Em_V}{E_F^2}\zeta_V^\parallel(E_F)\right],\\
A_0(q^2) &= \left(1-\frac{m_V^2}{2EM}\right)\frac{E}{\Delta}\zeta_V^\parallel(E_F),\\
T_1(q^2) &= \left(1-\frac{m_V^2}{2EM}\right)\frac{E_F}{\Delta}\zeta_V^\perp(E_F),\\
T_2(q^2) &= \frac{2E_F M}{M^2-m_V^2}\left(1-\frac{m_V^2}{2EM}\right)\zeta_V^\perp(E_F),\\
T_3(q^2) &= \frac{E_F^2}{\Delta^2}\left[\left(1+\frac{m_V^2}{E_F M}\right)\left(1-\frac{m_V^2}{2EM}\right)\zeta_V^\perp(E_F) - \left(\frac{M^2-m_V^2}{M^2}\right)\frac{Em_V}{E_F^2}\zeta_V^\parallel(E_F)\right].
\end{aligned} \tag{37}$$

Within the accuracy of our consideration we further expand expressions (37) up to terms of order m_V^2/M^2

$$\begin{aligned}
V(q^2) &= \frac{M+m_V}{M}\frac{E_F}{\Delta}\zeta_V^{\perp}(E_F),\\
A_1(q^2) &= \frac{2E_F}{M+m_V}\zeta_V^{\perp}(E_F),\\
A_2(q^2) &= \frac{M+m_V}{M}\left(1+\frac{2m_V^2}{M^2}\right)\left[\zeta_V^{\perp}(E_F)-\frac{m_V}{E_F}\zeta_V^{\parallel}(E_F)\right],\\
A_0(q^2) &= \zeta_V^{\parallel}(E_F),\\
T_1(q^2) &= \left(1-\frac{m_V^2}{M^2}\right)\frac{E_F}{\Delta}\zeta_V^{\perp}(E_F),\\
T_2(q^2) &= \frac{2E_F}{M}\zeta_V^{\perp}(E_F),\\
T_3(q^2) &= \left(1+\frac{5m_V^2}{M^2}\right)\zeta_V^{\perp}(E_F)-\left(1+\frac{2m_V^2}{M^2}\right)\frac{m_V}{E_F}\zeta_V^{\parallel}(E_F).
\end{aligned} \tag{38}$$

From Eqs. (38) it is possible to deduce the following relations between the heavy-to-light form factor for the case of the final vector meson

$$\begin{aligned}
\frac{M}{M+m_V}\frac{\Delta}{E_F}V(q^2) &= \frac{M+m_V}{2E_F}A_1(q^2) = \frac{M}{M+m_V}\left(1-\frac{2m_V^2}{M^2}\right)A_2(q^2)+\frac{m_V}{E_F}A_0(q^2)\\
&= \left(1+\frac{m_V^2}{M^2}\right)\frac{\Delta}{E_F}T_1(q^2) = \frac{M}{2E_F}T_2(q^2) = \zeta_V^{\perp}(E_F),
\end{aligned} \tag{39}$$

$$\begin{aligned}
\frac{m_V}{E_F}A_0(q^2) &= \frac{M+m_V}{2E_F}A_1(q^2) - \frac{M}{M+m_V}\left(1-\frac{2m_V^2}{M^2}\right)A_2(q^2)\\
&= \left(1+\frac{2m_V^2}{M^2}\right)T_1(q^2)-\left(1-\frac{2m_V^2}{M^2}\right)T_3(q^2)\\
&= \frac{M}{2E_F}\left(1+\frac{3m_V^2}{M^2}\right)T_2(q^2)-\left(1-\frac{2m_V^2}{M^2}\right)T_3(q^2) = \frac{m_V}{E_F}\zeta_V^{\parallel}(E_F).
\end{aligned} \tag{40}$$

These relations differ by terms of second order in m_V/E_F from the corresponding relations of Refs. [9,11] where only some of m_V^2/E_F^2 terms were taken into account. In particular Eq. (39) leads to the ratios of form factors $V(q^2)$ to $A_1(q^2)$ and $T_1(q^2)$ to $T_2(q^2)$, which depend only on meson masses and energies

$$\begin{aligned}
\frac{V(q^2)}{A_1(q^2)} &= \frac{(M+m_V)^2}{2M\Delta} = \frac{(M+m_V)^2}{2M\sqrt{E_F^2-m_V^2}},\\
\frac{T_1(q^2)}{T_2(q^2)} &= \frac{M^2-m_V^2}{2M\Delta} = \frac{M^2-m_V^2}{2M\sqrt{E_F^2-m_V^2}}.
\end{aligned} \tag{41}$$

Note that the complete account of the m_V^2/E_F^2 corrections results in the replacement of E_F by $\Delta=\sqrt{E_F^2-m_V^2}$ in the analogous relation (109) of Ref. [9]. Equations (41) ensure exact vanishing of the transverse helicity $\lambda = +1$ contribution to the rates of $B \to Ve\nu$ and $B \to Vl\bar{l}$ decays, since the corresponding helicity amplitudes are given by

$$H_{\pm}^{B\to Ve\nu}(q^2) = \frac{2M\Delta}{M+m_V}\left[V(q^2) \mp \frac{(M+m_V)^2}{2M\Delta}A_1(q^2)\right],$$
$$H_{\pm}^{B\to Vl\bar{l}}(q^2) = 2M\Delta\left[T_1(q^2) \mp \frac{M^2-m_V^2}{2M\Delta}T_2(q^2)\right]. \tag{42}$$

Such behaviour of helicity amplitudes is the consequence of the $(V-A)$ structure of weak currents in the standard model, which results in the creation of a left-handed light quark in the ultra relativistic limit, and the helicity-flip amplitude is suppressed by the factor $\Lambda_{\rm QCD}/E_F$ [13]. Therefore the helicity $\lambda = -1$ amplitude, e. g. for $B \to Ve\nu$ decay, can be expressed in the form

$$H_{-}^{B\to Ve\nu}(q^2) = 2(M+m_V)A_1(q^2). \tag{43}$$

The helicity $\lambda = 0$ amplitude for this decay is given by

$$H_0^{B\to Ve\nu}(q^2) = \frac{1}{2m_V\sqrt{q^2}}\left[(M+m_V)(M^2-m_V^2-q^2)A_1(q^2) - \frac{4M^2\Delta^2}{M+m_V}A_2(q^2)\right] \tag{44}$$

and can be rewritten with the account of symmetry relations (37) in a more simple form

$$H_0^{B\to Ve\nu}(q^2) = \frac{2M\Delta}{\sqrt{q^2}}A_0(q^2). \tag{45}$$

Then the ratio of helicity $\lambda = -1$ and $\lambda = 0$ amplitudes reads

$$\frac{|H_-(q^2)|}{|H_0(q^2)|} = \frac{M+m_V}{M}\frac{\sqrt{q^2}}{\Delta}\frac{A_1(q^2)}{A_0(q^2)} = \frac{2\sqrt{q^2}}{M-E_F+\Delta}\frac{E_F}{E}\frac{|\zeta_V^{\perp}(E_F)|}{|\zeta_V^{\parallel}(E_F)|}. \tag{46}$$

The Isgur-Wise relations [2,14,13] between the heavy-to-light form factors follow from the heavy-quark spin symmetry alone, and were obtained near the point of zero recoil ($\Delta \approx 0$, $q^2 \approx q^2_{\rm max} = (M-m_V)^2$) of the final light meson in the heavy quark limit

$$\begin{aligned}
&\frac{2M}{M+m_P}f_T(q^2) = f_+(q^2) + \frac{M^2-m_P^2}{q^2}\left[f_+(q^2) - f_0(q^2)\right],\\
&T_1(q^2) = \frac{M^2+q^2-m_V^2}{2M}\frac{V(q^2)}{M+m_V} + \frac{M+m_V}{2M}A_1(q^2),\\
&\frac{M^2-m_V^2}{q^2}\left[T_1(q^2)-T_2(q^2)\right] = \frac{3M^2-q^2+m_V^2}{2M}\frac{V(q^2)}{M+m_V} - \frac{M+m_V}{2M}A_1(q^2),\\
&T_3(q^2) = \frac{M^2-q^2+3m_V^2}{2M}\frac{V(q^2)}{M+m_V} + \frac{M^2-m_V^2}{Mq^2}m_V A_0(q^2)\\
&\qquad -\frac{M^2+q^2-m_V^2}{2Mq^2}\left[(M+m_V)A_1(q^2) - (M-m_V)A_2(q^2)\right].
\end{aligned} \tag{47}$$

They are exactly satisfied by form factors (25) and (37). Thus we conclude that these relations remain valid near the point $q^2 = 0$ corresponding to the maximum recoil of the final light meson if the large recoil limit is used in addition. Note that the Isgur-Wise relations (47) hold in these limits for B decays both to ground state vector (pseudoscalar) light mesons and their radial excitations.

Some heavy-to-light form factor relations for decays to pseudoscalar and vector mesons were obtained by Stech [7] and Soares [8] in the framework of the constituent quark model. It is easy to check that the additional relations found at the large recoil momentum of the final meson [8]

$$\begin{aligned} f_0(q^2) &= f_+(q^2)\left(1-\frac{q^2}{M^2-m_V^2}\frac{M+E_F-\Delta}{M-E_F+\Delta}\right), \\ V(q^2) &= \frac{(M+m_V)^2}{2M\Delta}A_1(q^2) = \frac{M}{ME_F-m_V^2}\left[\Delta A_2(q^2)+\frac{M+m_V}{M}m_V A_0(q^2)\right] \end{aligned} \tag{48}$$

are also satisfied exactly by form factors (25) and (37). The other relations [12] were derived in the spectator approximation only. Instead of them we get

$$\begin{aligned} \frac{E_F}{E}\frac{\zeta_V^\perp(E_F)}{\zeta_V^\parallel(E_F)} = \frac{T_1(q^2)}{A_0(q^2)} &= \frac{m_V(M-m_V)T_2(q^2)}{(ME_F-m_V^2)A_1(q^2)-[2M^2\Delta^2/(M+m_V)^2]\,A_2(q^2)} \\ &= \frac{m_V(M+m_V)A_1(q^2)}{(ME_F+m_V^2)T_2(q^2)-[2M^2\Delta^2/(M^2-m_V^2)]\,T_3(q^2)}. \end{aligned} \tag{49}$$

Thus their fulfilment in the form given in [12] requires the specific relation between invariant functions

$$\zeta_V^\parallel(E_F) = \frac{E_F}{E}\zeta_V^\perp(E_F). \tag{50}$$

IV. B DECAYS TO ORBITALLY EXCITED LIGHT MESONS

Now we investigate the heavy-to-light B decays to orbitally excited P-wave light mesons.

A. B decays to scalar light mesons

The matrix elements of the weak current for B decays to orbitally excited scalar light mesons can be parametrized by three invariant form factors

$$\langle S(p_F)|\bar q\gamma^\mu\gamma_5 b|B(p_B)\rangle = r_+(q^2)\,(p_B^\mu+p_F^\mu)+r_-(q^2)\,(p_B^\mu-p_F^\mu)\,, \tag{51}$$

$$\langle S(p_F)|\bar q\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle = \frac{ir_T(q^2)}{M+m_S}[q^2(p_B^\mu+p_F^\mu)-(M^2-m_P^2)q^\mu], \tag{52}$$

where m_S is the scalar meson mass.

The symmetry relations for the matrix elements can be obtained by calculating corresponding traces in Eq. (15)

$$\langle S(p_F)|\bar{q}\gamma^\mu\gamma_5 b|B(p_B)\rangle = 2E_F\zeta_S(E_F)n^\mu, \tag{53}$$

$$\langle S(p_F)|\bar{q}\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle = 2E_F\zeta_S(E_F)\left[(M-E_F)n^\mu - M\left(1-\frac{m_S^2}{2EM}v^\mu\right)\right]. \tag{54}$$

These equations show that the form factors can be parametrized by one invariant function only in the heavy quark and large recoil limit. Comparing Eqs. (51), (52) and (53), (54) we find

$$\begin{aligned}
r_+(q^2) &= \left(1-\frac{m_S^2}{2EM}\right)\frac{E_F}{\Delta}\zeta_S(E_F),\\
r_-(q^2) &= -\left(1+\frac{m_S^2}{2EM}\right)\frac{E_F}{\Delta}\zeta_S(E_F),\\
r_T(q^2) &= \frac{M+m_S}{M}\frac{E_F}{\Delta}\zeta_S(E_F).
\end{aligned} \tag{55}$$

Expanding Eqs. (55) in m_S/E, we get retaining the terms of order m_S^2/M^2

$$\begin{aligned}
r_+(q^2) &= \left(1+\frac{m_S^2}{M^2}\right)\zeta_S(E_F),\\
r_-(q^2) &= -\left(1+\frac{3m_S^2}{M^2}\right)\zeta_S(E_F),\\
r_T(q^2) &= \frac{M+m_S}{M}\left(1+\frac{2m_S^2}{M^2}\right)\zeta_S(E_F).
\end{aligned} \tag{56}$$

These expressions for the form factors lead to the following symmetry relations between them

$$r_+(q^2) = -\left(1-\frac{2m_S^2}{M^2}\right)r_-(q^2) = \frac{M}{M+m_S}\left(1-\frac{m_S^2}{M^2}\right)r_T(q^2) = \left(1+\frac{m_S^2}{M^2}\right)\zeta_S(E_F). \tag{57}$$

B. B decays to axial vector light mesons

The matrix elements of the weak current for B decays to axial vector mesons can be expressed in terms of seven invariant form factors

$$\langle A(p_F)|\bar{q}\gamma^\mu b|B(p_B)\rangle = (M+m_A)t_{V_1}(q^2)\epsilon^{*\mu} + [t_{V_2}(q^2)p_B^\mu + t_{V_3}(q^2)p_F^\mu]\frac{\epsilon^*\cdot q}{M}, \tag{58}$$

$$\langle A(p_F)|\bar{q}\gamma^\mu\gamma_5 b|B(p_B)\rangle = \frac{2it_A(q^2)}{M+m_A}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_{B\rho}p_{F\sigma} \tag{59}$$

$$\begin{aligned}
\langle A(p_F)|\bar{q}i\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = {}& t_+(q^2)\left[(\epsilon^*\cdot q)(p_B+p_F)^\mu - \epsilon^{*\mu}(M^2-m_A^2)\right]\\
&+t_-(q^2)\left[(\epsilon^*\cdot q)q^\mu - \epsilon^{*\mu}q^2\right]
\end{aligned}$$

$$+t_0(q^2)\frac{\epsilon^*\cdot q}{M^2}\left[(M^2-m_A^2)q^\mu-q^2(p_B+p_F)^\mu\right], \tag{60}$$

$$\langle A(p_F)|\bar{q}i\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle = 2it_+(q^2)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu p_{B\rho}p_{F\sigma}, \tag{61}$$

where m_A and ϵ^μ are the mass and polarization vector of the axial vector meson.

Equation (15) yields the following symmetry equations in this case

$$\begin{aligned}\langle A(p_F)|\bar{q}\gamma^\mu b|B(p_B)\rangle = 2E_F\Big\{&\zeta_A^\perp(E_F)\left[\epsilon^{*\mu}-\epsilon^*\cdot v\left(\frac{E_F}{\Delta}n^\mu-\frac{m_A^2}{2E\Delta}v^\mu\right)\right]\\ &+\frac{E}{\Delta}\frac{m_A}{E_F}\zeta_A^\parallel(E_F)\epsilon^*\cdot v n^\mu\Big\},\end{aligned} \tag{62}$$

$$\langle A(p_F)|\bar{q}\gamma^\mu\gamma_5 b|B(p_B)\rangle = 2iE_F\zeta_A^\perp(E_F)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu v_\rho n_\sigma, \tag{63}$$

$$\begin{aligned}\langle A(p_F)|\bar{q}i\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = -2E_F\Big\{&M\zeta_A^\perp(E_F)\left(1-\frac{m_A^2}{2EM}\right)\Big[\epsilon^{*\mu}-\epsilon^*\cdot v\Big(\frac{E_F}{\Delta}n^\mu\\ &-\frac{m_A^2}{2E\Delta}v^\mu\Big)\Big]+\frac{E}{\Delta}\frac{m_A}{E_F}\zeta_A^\parallel(E_F)\epsilon^*\cdot v\Big[(M-E_F)n^\mu\\ &-M\left(1-\frac{m_A^2}{2EM}\right)v^\mu\Big]\Big\},\end{aligned} \tag{64}$$

$$\langle A(p_F)|\bar{q}i\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle = 2iE_F M\zeta_A^\perp(E_F)\left(1-\frac{m_A^2}{2EM}\right)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu v_\rho n_\sigma. \tag{65}$$

Thus all form factors for B decays to axial vector mesons in the heavy quark and large recoil limit can be expressed through two invariant functions

$$\begin{aligned}t_A(q^2) &= \frac{M+m_A}{M}\frac{E_F}{\Delta}\zeta_A^\perp(E_F),\\ t_{V_1}(q^2) &= \frac{2E_F}{M+m_A}\zeta_A^\perp(E_F),\\ t_{V_2}(q^2) &= \frac{E_F m_A^2}{\Delta^2 M}\left[2\zeta_A^\perp(E_F)-\frac{m_A}{E_F}\zeta_A^\parallel(E_F)\right],\\ t_{V_3}(q^2) &= 2\frac{E_F^2}{\Delta^2}\left[\frac{Em_A}{E_F^2}\zeta_A^\parallel(E_F)-\zeta_A^\perp(E_F)\right],\\ t_+(q^2) &= \left(1-\frac{m_A^2}{2EM}\right)\frac{E_F}{\Delta}\zeta_A^\perp(E_F),\\ t_-(q^2) &= -\left(1+\frac{m_A^2}{2EM}\right)\frac{E_F}{\Delta}\zeta_A^\perp(E_F),\\ t_0(q^2) &= \frac{E_F E}{\Delta^2}\left[\frac{m_A}{E_F}\zeta_A^\parallel(E_F)-\frac{m_A^2}{2E^2}\zeta_A^\perp(E_F)\right].\end{aligned} \tag{66}$$

Keeping the terms of order m_A^2/M^2 in Eqs. (66) we find

$$t_A(q^2) = \frac{M+m_A}{M}\frac{E_F}{\Delta}\zeta_A^\perp(E_F),$$

$$
\begin{aligned}
t_{V_1}(q^2) &= \frac{2E_F}{M+m_A}\zeta_A^{\perp}(E_F), \\
t_{V_2}(q^2) &= \frac{2m_A^2}{M^2}\left[2\zeta_A^{\perp}(E_F) - \frac{m_A}{E_F}\zeta_A^{\parallel}(E_F)\right], \\
t_{V_3}(q^2) &= 2\left(1+\frac{4m_A^2}{M^2}\right)\left[\left(1-\frac{m_A^2}{M^2}\right)\frac{m_A}{E_F}\zeta_A^{\parallel}(E_F) - \zeta_A^{\perp}(E_F)\right], \\
t_+(q^2) &= \left(1-\frac{m_A^2}{M^2}\right)\frac{E_F}{\Delta}\zeta_A^{\perp}(E_F), \\
t_-(q^2) &= -\left(1+\frac{m_A^2}{M^2}\right)\frac{E_F}{\Delta}\zeta_A^{\perp}(E_F), \\
t_0(q^2) &= \left(1+\frac{3m_A^2}{M^2}\right)\frac{m_A}{E_F}\zeta_A^{\parallel}(E_F) - \frac{2m_A^2}{M^2}\zeta_A^{\perp}(E_F).
\end{aligned} \tag{67}
$$

On the basis of Eqs. (67) the following symmetry relations between the form factors can be derived

$$
\begin{aligned}
\frac{M}{M+m_A}\frac{\Delta}{E_F}t_A(q^2) &= \frac{M+m_A}{2E_F}t_{V_1}(q^2) = \left(1+\frac{m_A^2}{M^2}\right)\frac{\Delta}{E_F}t_+(q^2) \\
&= -\left(1-\frac{m_A^2}{M^2}\right)\frac{\Delta}{E_F}t_-(q^2) = \zeta_A^{\perp}(E_F),
\end{aligned} \tag{68}
$$

$$
\begin{aligned}
&\frac{1}{2}\left(1-\frac{3m_A^2}{M^2}\right)t_{V_3}(q^2) + \left(1+\frac{m_A^2}{M^2}\right)\frac{M+m_A}{2E_F}t_{V_1}(q^2) \\
&\qquad = \left(1-\frac{3m_A^2}{M^2}\right)t_0(q^2) + \frac{2m_A^2}{M^2}t_+(q^2) = \frac{m_A}{E_F}\zeta_A^{\parallel}(E_F),
\end{aligned} \tag{69}
$$

$$
t_{V_2}(q^2) = -\frac{m_A^2}{M^2}\left[t_{V_3}(q^2) - 2t_{V_1}(q^2)\right].
$$

Of particular interest are the ratios of form factors which depend only on meson masses and recoil momentum following from Eqs. (68), (69)

$$
\begin{aligned}
\frac{t_A(q^2)}{t_{V_1}(q^2)} &= \frac{(M+m_A)^2}{2M\Delta}, \\
\frac{t_+(q^2)}{t_+(q^2) + [q^2/(M^2-m_A^2)]t_-(q^2)} &= \frac{M^2-m_A^2}{2M\Delta}.
\end{aligned} \tag{70}
$$

These relations correspond to the exact vanishing of helicity amplitudes $H_+(q^2)$ in the heavy quark and large recoil limit. This is similar to the case of B decays to vector light mesons. With the account of equations (70), the ratio of helicity $\lambda = -1$ and $\lambda = 0$ amplitude takes the form

$$
\frac{|H_-(q^2)|}{|H_0(q^2)|} = \frac{2\sqrt{q^2}}{M - E_F + \Delta}\frac{E_F}{E}\frac{|\zeta_A^{\perp}(E_F)|}{|\zeta_A^{\parallel}(E_F)|}. \tag{71}
$$

C. B decays to tensor light mesons

The matrix elements of weak current for B decays to tensor mesons can be decomposed in seven Lorentz-invariant structures

$$\langle T(p_F)|\bar q\gamma^\mu b|B(p_B)\rangle = \frac{2ig_V(q^2)}{M+m_T}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_{\nu\alpha}\frac{p_B^\alpha}{M}p_{B\rho}p_{F\sigma}, \tag{72}$$

$$\begin{aligned}\langle T(p_F)|\bar q\gamma^\mu\gamma_5 b|B(p_B)\rangle &= (M+m_T)g_{A_1}(q^2)\epsilon^{*\mu\alpha}\frac{p_{B\alpha}}{M}\\ &+[g_{A_2}(q^2)p_B^\mu + g_{A_3}(q^2)p_F^\mu]\epsilon^*_{\alpha\beta}\frac{p_B^\alpha p_B^\beta}{M^2}\end{aligned} \tag{73}$$

$$\langle T(p_F)|\bar q i\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = 2ig_+(q^2)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_{\nu\alpha}\frac{p_{B\alpha}}{M}p_{B\rho}p_{F\sigma}, \tag{74}$$

$$\begin{aligned}\langle T(p_F)|\bar q i\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle &= g_+(q^2)\left[\epsilon^*_{\alpha\beta}\frac{p_B^\alpha p_B^\beta}{M}(p_B+p_F)^\mu - \epsilon^{*\mu\alpha}\frac{p_{B\alpha}}{M}(M^2-m_T^2)\right]\\ &+g_-(q^2)\left[\epsilon^*_{\alpha\beta}\frac{p_B^\alpha p_B^\beta}{M}q^\mu - \epsilon^{*\mu\alpha}\frac{p_{B\alpha}}{M}q^2\right]\\ &+g_0(q^2)\epsilon^*_{\alpha\beta}\frac{p_B^\alpha p_B^\beta}{M^3}\left[(M^2-m_T^2)q^\mu - q^2(p_B+p_F)^\mu\right],\end{aligned} \tag{75}$$

where m_T and $\epsilon^{\mu\nu}$ are the mass and polarization tensor of the tensor meson.

Calculating the trace in Eq. (15) with the account of (19) and (13), (14) in the heavy quark and large recoil limit, we get

$$\langle T(p_F)|\bar q\gamma^\mu b|B(p_B)\rangle = 2iE_F\zeta_T^\perp(E_F)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_{\nu\alpha}v^\alpha v_\rho n_\sigma, \tag{76}$$

$$\begin{aligned}\langle T(p_F)|\bar q\gamma^\mu\gamma_5 b|B(p_B)\rangle &= 2E_F\Bigg\{\zeta_T^\perp(E_F)\left[\epsilon^{*\mu\alpha}v_\alpha - \epsilon^*_{\alpha\beta}v^\alpha v^\beta\left(\frac{E_F}{\Delta}n^\mu - \frac{m_T^2}{2E\Delta}v^\mu\right)\right]\\ &+\frac{E}{\Delta}\frac{m_T}{E_F}\zeta_T^\parallel(E_F)\epsilon^*_{\alpha\beta}v^\alpha v^\beta n^\mu\Bigg\},\end{aligned} \tag{77}$$

$$\langle T(p_F)|\bar q i\sigma^{\mu\nu}q_\nu b|B(p_B)\rangle = 2iE_F M\zeta_T^\perp(E_F)\left(1-\frac{m_T^2}{2EM}\right)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_{\nu\alpha}v^\alpha n_\rho v_\sigma, \tag{78}$$

$$\begin{aligned}\langle T(p_F)|\bar q i\sigma^{\mu\nu}\gamma_5 q_\nu b|B(p_B)\rangle &= 2E_F\Bigg\{M\zeta_T^\perp(E_F)\left(1-\frac{m_T^2}{2EM}\right)\Bigg[\epsilon^{*\mu\alpha}v_\alpha - \epsilon^*_{\alpha\beta}v^\alpha v^\beta\Bigg(\frac{E_F}{\Delta}n^\mu\\ &-\frac{m_T^2}{2E\Delta}v^\mu\Bigg)\Bigg] + \frac{E}{\Delta}\frac{m_T}{E_F}\zeta_T^\parallel(E_F)\epsilon^*_{\alpha\beta}v^\alpha v^\beta\Bigg[(M-E_F)n^\mu\\ &-M\left(1-\frac{m_T^2}{2EM}\right)v^\mu\Bigg]\Bigg\}.\end{aligned} \tag{79}$$

As in the case of vector and axial vector final mesons all form factors for B decays to tensor mesons can be expressed in terms of two invariant functions

$$g_V(q^2) = \frac{M+m_T}{M}\frac{E_F}{\Delta}\zeta_T^\perp(E_F),$$

$$
\begin{aligned}
g_{A_1}(q^2) &= \frac{2E_F}{M+m_T}\zeta_T^{\perp}(E_F), \\
g_{A_2}(q^2) &= \frac{E_F m_T^2}{\Delta^2 M}\left[2\zeta_T^{\perp}(E_F) - \frac{m_T}{E_F}\zeta_T^{\parallel}(E_F)\right], \\
g_{A_3}(q^2) &= 2\frac{E_F^2}{\Delta^2}\left[\frac{E m_T}{E_F^2}\zeta_T^{\parallel}(E_F) - \zeta_T^{\perp}(E_F)\right], \\
g_{+}(q^2) &= -\left(1-\frac{m_T^2}{2EM}\right)\frac{E_F}{\Delta}\zeta_T^{\perp}(E_F), \\
g_{-}(q^2) &= \left(1+\frac{m_T^2}{2EM}\right)\frac{E_F}{\Delta}\zeta_T^{\perp}(E_F), \\
g_{0}(q^2) &= \frac{E_F E}{\Delta^2}\left[\frac{m_T^2}{2E^2}\zeta_T^{\perp}(E_F) - \frac{m_T}{E_F}\zeta_T^{\parallel}(E_F)\right].
\end{aligned} \tag{80}
$$

Performing the m_T/E expansion of Eqs. (80), we obtain up to terms of order m_T^2/M^2

$$
\begin{aligned}
g_V(q^2) &= \frac{M+m_T}{M}\frac{E_F}{\Delta}\zeta_T^{\perp}(E_F), \\
g_{A_1}(q^2) &= \frac{2E_F}{M+m_T}\zeta_T^{\perp}(E_F), \\
g_{A_2}(q^2) &= \frac{2m_T^2}{M^2}\left(2\zeta_T^{\perp}(E_F) - \frac{m_T}{E_F}\zeta_T^{\parallel}(E_F)\right), \\
g_{A_3}(q^2) &= 2\left(1+\frac{4m_T^2}{M^2}\right)\left[\frac{m_T}{E_F}\left(1-\frac{m_T^2}{M^2}\right)\zeta_T^{\parallel}(E_F) - \zeta_T^{\perp}(E_F)\right], \\
g_{+}(q^2) &= -\left(1-\frac{m_T^2}{M^2}\right)\frac{E_F}{\Delta}\zeta_T^{\perp}(E_F), \\
g_{-}(q^2) &= \left(1+\frac{m_T^2}{M^2}\right)\frac{E_F}{\Delta}\zeta_T^{\perp}(E_F), \\
g_{0}(q^2) &= -\left(1+\frac{3m_T^2}{M^2}\right)\frac{m_T}{E_F}\zeta_T^{\parallel}(E_F) + \frac{2m_T^2}{M^2}\zeta_T^{\perp}(E_F).
\end{aligned} \tag{81}
$$

As a result the following form factor relations arise from Eqs. (81)

$$
\begin{aligned}
\frac{M}{M+m_T}\frac{\Delta}{E_F}g_V(q^2) &= \frac{M+m_T}{2E_F}g_{A_1}(q^2) = -\left(1+\frac{m_T^2}{M^2}\right)\frac{\Delta}{E_F}g_{+}(q^2) \\
&= \left(1-\frac{m_T^2}{M^2}\right)\frac{\Delta}{E_F}g_{-}(q^2) = \zeta_T^{\perp}(E_F),
\end{aligned} \tag{82}
$$

$$
\begin{aligned}
&\frac{1}{2}\left(1-\frac{3m_T^2}{M^2}\right)g_{A_3}(q^2) + \left(1+\frac{m_T^2}{M^2}\right)\frac{M+m_T}{2E_F}g_{A_1}(q^2) \\
&\qquad = -\left(1-\frac{3m_T^2}{M^2}\right)g_0(q^2) - \frac{2m_T^2}{M^2}g_{+}(q^2) = \frac{m_T}{E_F}\zeta_T^{\parallel}(E_F),
\end{aligned} \tag{83}
$$

$$
g_{A_2}(q^2) = -\frac{m_T^2}{M^2}\left[g_{A_3}(q^2) - 2g_{A_1}(q^2)\right].
$$

Two particular ratios of form factors

$$\frac{g_V(q^2)}{g_{A_1}(q^2)} = \frac{(M+m_A)^2}{2M\Delta},$$
$$\frac{g_+(q^2)}{g_+(q^2)+[q^2/(M^2-m_A^2)]g_-(q^2)} = \frac{M^2-m_A^2}{2M\Delta} \tag{84}$$

again lead to the exact vanishing of the corresponding helicity amplitude $H_+(q^2)$ in the heavy quark and large recoil limit. Similarly the ratio of helicity $\lambda = -1$ and $\lambda = 0$ amplitudes is given by

$$\frac{|H_-(q^2)|}{|H_0(q^2)|} = \frac{2\sqrt{q^2}}{M-E_F+\Delta}\frac{E_F}{E}\frac{|\zeta_T^\perp(E_F)|}{|\zeta_T^\parallel(E_F)|}. \tag{85}$$

V. SYMMETRY RELATIONS IN THE RELATIVISTIC QUARK MODEL

We can test the fulfilment of the symmetry relations for the form factors, arising at large recoil of the final meson, in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. In Refs. [6] and [5] we considered exclusive semileptonic and rare radiative B decays to ground state light mesons. The analysis of these decays was performed near the point of the maximum recoil of the final light meson employing the expansions both in inverse powers of the heavy b-quark mass in the initial state and in the inverse large recoil momentum of the final pseudoscalar or vector meson. The resulting expressions are valid up to the second order terms in these expansions and for $q^2 = 0$. The general formulas (A1), (A4), (A7), (A10) from Ref. [6] and (24) from Ref. [5] can be applied for decays to ground state light mesons as well as for their radial excitations. Keeping only the leading terms in $\Lambda_{\rm QCD}/m_b$, $\Lambda_{\rm QCD}/E_F$ and terms quadratic in m/E_F we compare the resulting expressions with symmetry relations (28), (39), (40). It is easy to check that the symmetry relations between the form factors are satisfied in our relativistic quark model. As a result one can determine corresponding invariant functions. In this way we get:

(i) for B decays to pseudoscalar light mesons

$$\zeta_P(E_F) = \sqrt{\frac{M}{2E_F}}\int\frac{d^3p}{(2\pi)^3}\bar\psi_P\left(\mathbf{p}+\frac{2\epsilon_q}{E_F+m_P}\mathbf{\Delta}\right)\psi_B(\mathbf{p}), \tag{86}$$

(ii) for B decays to vector light mesons

$$\zeta_V^\perp(E_F) = \sqrt{\frac{M}{2E_F}}\int\frac{d^3p}{(2\pi)^3}\bar\psi_V\left(\mathbf{p}+\frac{2\epsilon_q}{E_F+m_V}\mathbf{\Delta}\right)\psi_B(\mathbf{p}), \tag{87}$$
$$\zeta_V^\parallel(E_F) = \frac{E_F}{E}\zeta_V^\perp(E_F). \tag{88}$$

Since Eq. (88) coincides with Eq. (50), we can rewrite the relations (49) in the form:

$$T_1(q^2) = A_0(q^2),$$

$$m_V(M-m_V)T_2(q^2) = (ME_F - m_V^2)A_1(q^2) - \frac{2M^2\Delta^2}{(M+m_V)^2}A_2(q^2),$$

$$(ME_F + m_V^2)T_2(q^2) - \frac{2M^2\Delta^2}{(M^2-m_V^2)}T_3(q^2) = m_V(M+m_V)A_1(q^2), \tag{89}$$

thus recovering the form factor relations of Ref. [12] near the maximum recoil of the final vector meson.

In Ref. [15] we considered rare radiative B decays to orbitally excited K mesons up to the second order terms of heavy quark and large recoil momentum expansions. We further retain only the contributions of leading order in $\Lambda_{\rm QCD}/m_b$, $\Lambda_{\rm QCD}/E_F$ and of second order in m/E_F in Eqs. (29), (33) and (37) of Ref. [15]. Then we perform a similar analysis for the semileptonic B decays to orbitally excited scalar, axial vector and tensor light mesons. The symmetry relations (57), (68), (69), (82), (83) are valid in our model and invariant functions can be obtained. Thus we find:

(*iii*) for B decays to scalar light mesons

$$\zeta_S(E_F) = \frac{\Delta}{E_F+m_S}\frac{1}{3}\sqrt{\frac{M}{2E_F}}\int\frac{d^3p}{(2\pi)^3}\bar\psi_S\Big(\mathbf{p}+\frac{2\epsilon_q}{E_F+m_S}\mathbf{\Delta}\Big)$$
$$\times\left[-3(E_F+m_S)\frac{\mathbf{p}\cdot\mathbf{\Delta}}{p\Delta^2} - \frac{2p}{\epsilon_q(p)+m_q}\right]\psi_B(\mathbf{p}), \tag{90}$$

(*iv*) for B decays to axial vector light meson with $j=1/2$

$$\zeta^\perp_{A(1/2)}(E_F) = \frac{\Delta}{E_F+m_{A(1/2)}}\frac{1}{3}\sqrt{\frac{M}{2E_F}}\int\frac{d^3p}{(2\pi)^3}\bar\psi_{A(1/2)}\Big(\mathbf{p}+\frac{2\epsilon_q}{E_F+m_{A(1/2)}}\mathbf{\Delta}\Big)$$
$$\times\left[-3(E_F+m_{A(1/2)})\frac{\mathbf{p}\cdot\mathbf{\Delta}}{p\Delta^2} - \frac{2p}{\epsilon_q(p)+m_q}\right]\psi_B(\mathbf{p}), \tag{91}$$

$$\zeta^\parallel_{A(1/2)}(E_F) = -\frac{E_F}{E}\zeta^\perp_{A(1/2)}(E_F), \tag{92}$$

(*v*) for B decays to axial vector light mesons with $j=3/2$

$$\zeta^\perp_{A(3/2)}(E_F) = \frac{\Delta}{E_F+m_{A(3/2)}}\frac{1}{3\sqrt2}\sqrt{\frac{M}{2E_F}}\int\frac{d^3p}{(2\pi)^3}\bar\psi_{A(3/2)}\Big(\mathbf{p}+\frac{2\epsilon_q}{E_F+m_{A(3/2)}}\mathbf{\Delta}\Big)$$
$$\times\left[-3(E_F+m_{A(3/2)})\frac{\mathbf{p}\cdot\mathbf{\Delta}}{p\Delta^2} + \frac{p}{\epsilon_q(p)+m_q}\right]\psi_B(\mathbf{p}), \tag{93}$$

$$\zeta^\parallel_{A(3/2)}(E_F) = 2\frac{E_F}{E}\zeta^\perp_{A(3/2)}(E_F), \tag{94}$$

(*vi*) for B decays to tensor light mesons

$$\zeta^\perp_T(E_F) = \frac{m_T}{E_F+m_T}\frac{1}{\sqrt3}\sqrt{\frac{M}{2E_F}}\int\frac{d^3p}{(2\pi)^3}\bar\psi_T\Big(\mathbf{p}+\frac{2\epsilon_q}{E_F+m_T}\mathbf{\Delta}\Big)$$

$$\times \left[-3(E_F + m_T)\frac{\mathbf{p}\cdot\mathbf{\Delta}}{p\Delta^2} + \frac{p}{\epsilon_q(p)+m_q}\right]\psi_B(\mathbf{p}), \tag{95}$$

$$\zeta_T^{\parallel}(E_F) = \frac{E_F}{E}\zeta_T^{\perp}(E_F). \tag{96}$$

Here ψ_B and ψ_F are the B meson and light meson wave functions. We used the HQET js-coupling scheme for parametrizing axial vector state wave functions, which is convenient for the description of axial vector orbital excitations of K mesons [15]. Instead, for axial vector excitations of π, ρ mesons the LS-coupling scheme should be used where ${}^1P_1$ and ${}^3P_1$ states are the linear combinations of $A(1/2)$ and $A(3/2)$ states.

The ratio of the helicity $\lambda = -1$ and $\lambda = 0$ amplitudes (46), (71), (85) with the account of quark model relations (88), (92), (94), (96) for small q^2 will depend on meson masses and energies only

$$\frac{|H_-(q^2)|}{|H_0(q^2)|} = \frac{2\sqrt{M^2+m_V^2-2ME_F}}{M-E_F+\Delta}\begin{cases} 1 & F=V \\ 1 & F=A(1/2) \\ 1/2 & F=A(3/2) \\ 1 & F=T \end{cases}. \tag{97}$$

The ratio of the helicity $\lambda = +1$ and $\lambda = 0$ amplitudes vanishes, $|H_+(q^2)|/|H_0(q^2)| = 0$, for all above decays in the heavy quark and large recoil limit. These predictions can be tested in charmless nonleptonic B decays to two light final mesons in the factorization approximation.

In Tables I and II we give our predictions for the values of invariant functions $\zeta_{P,S}$, $\zeta_{V,A,T}^{\perp}$ and $\zeta_{V,A,T}^{\parallel}$ parametrizing B decays to light meson states at the maximum recoil of the final meson where $E_F^{\max} = (M^2+m^2)/(2M)$ and $q^2 = 0$. These values can be used for evaluating the decay rates.

Recently, in Refs. [16,17] the exclusive $B \to K^*\gamma$ decay branching fractions were calculated with the complete account of next-to-leading order QCD effects and to leading order in the heavy quark and large recoil limit. A sizable enhancement of the coefficient $\mathcal{C}_7$ at next-to-leading order was observed. Ref. [16] gives

$$|\mathcal{C}_7|^2_{\rm NLO}/|\mathcal{C}_7|^2_{\rm LO} \approx 1.78, \tag{98}$$

and the close value 1.6 was found in Ref. [17]. The decay branching fraction in the above limit should be proportional to the square of the transverse invariant function $\zeta_{K^*}^{\perp}(E_F^{\max})$. In order to evaluate $\zeta_{K^*}^{\perp}(E_F^{\max})$ the predictions of QCD sum rules [18] for the decay form factors $T_1^{K^*}(0)$ and $V^{K^*}(0)$ were used. The estimate of Ref. [16] yields $\zeta_{K^*}^{\perp}(E_F^{\max}) = 0.35 \pm 0.07$, which leads to the branching fraction

$$BR(B \to K^*\gamma) = (7.9^{+1.8}_{-1.6}) \times 10^{-5}\left(\frac{\tau_B}{1.6\ ps}\right)\left(\frac{m_b}{4.6\ \mathrm{GeV}}\right)^2\left(\frac{\zeta_{K^*}^{\perp}(E_F^{\max})}{0.35}\right)^2. \tag{99}$$

This value is considered in [16] to be nearly twice as large as the current experimental data

$$BR(B^0 \to K^{*0}\gamma) = \begin{cases} (4.55 \pm 0.70 \pm 0.34) \times 10^{-5} & \text{CLEO [19]} \\ (4.96 \pm 0.67 \pm 0.45) \times 10^{-5} & \text{Belle [20]} \\ (4.39 \pm 0.41 \pm 0.27) \times 10^{-5} & \text{BABAR [21]} \end{cases} \tag{100}$$

$$BR(B^+ \to K^{*+}\gamma) = \begin{cases} (3.76 \pm 0.86 \pm 0.28) \times 10^{-5} & \text{CLEO [19]} \\ (3.89 \pm 0.93 \pm 0.41) \times 10^{-5} & \text{Belle [20]} \end{cases}. \tag{101}$$

In Ref. [17] the value $F_{K^*}(0) = \zeta_{K^*}^{\perp}(E_F^{\max}) = 0.38 \pm 0.06$ is used and the estimates

$$BR(B^- \to K^{*-}\gamma) = 7.45 \times 10^{-5}, \qquad BR(B^0 \to K^{*0}\gamma) = 7.09 \times 10^{-5} \tag{102}$$

are obtained for central input parameters. The authors of [17] claim, that these results are compatible with experimental measurements taking the sizable uncertainties into account, even though the central theoretical values appear to be somewhat high.

However, $\zeta_{K^*}^{\perp}(E_F^{\max})$ determined from QCD sum rules contains $\Lambda_{\rm QCD}/m_b$ and $\Lambda_{\rm QCD}/E_F$ corrections which were neglected in deriving (98). As our model estimates show, $1/m_b$ corrections can give contributions of $10-15\%$ to the form factors. Indeed, with the account of such corrections we have $T_1^{K^*}(0) = 0.32(3)$ in our model [5] while in the large recoil limit $T_1^{K^*}(0) \to \zeta_{K^*}^{\perp}(E_F^{\max}) = 0.28$ (see Table I). If we substitute the latter value in Eq. (99), we get

$$BR(B \to K^*\gamma) = (5.0^{+1.8}_{-1.7}) \times 10^{-5}, \tag{103}$$

while the branching fractions (102) would be given in our model by

$$BR(B^- \to K^{*-}\gamma) = (4.1^{+1.5}_{-1.2}) \times 10^{-5}, \qquad BR(B^0 \to K^{*0}\gamma) = (3.9^{+1.5}_{-1.2}) \times 10^{-5} \tag{104}$$

in agreement with experimental data (100) and (101).

The similar substitution of our prediction for $\zeta_{\rho}^{\perp}(E_F^{\max}) = 0.24$ (see Table I) instead of the QCD sum rule [18] value $T_1^{\rho}(0) = 0.29 \pm 0.04$ in the expression for $BR(B \to \rho\gamma)$ with the complete account of next-to-leading order QCD corrections in the heavy quark and large recoil limit [17] (see also [22]) yields

$$BR(B^- \to \rho^-\gamma) = (1.1^{+0.5}_{-0.4}) \times 10^{-6}. \tag{105}$$

VI. CONCLUSIONS

In this paper we derived the form factor relations for B decays to light mesons arising in the heavy quark and large recoil energy limits. The decays both to ground state and radially and orbitally excited light mesons were considered. The main attention was devoted to the complete accounting for corrections of second order in the ratio of the light meson mass to the large recoil energy. Such corrections are especially important for decays to excited light mesons, since their masses are of order of the charmed quark mass. The correction to the effective Lagrangian quadratic in the final meson mass has been obtained. It was found that this correction does not violate the symmetry of the leading order Lagrangian, since it has the same Dirac structure as the leading contribution. Therefore the inclusion of corrections, which are quadratic in the ratio of the final meson mass to the large recoil energy, to the weak current matrix elements does not lead to the introduction of additional invariant functions. Their inclusion requires a more accurate consideration of the decay kinematics, keeping all final meson mass contributions. The heavy quark and large recoil symmetries substantially constrain the number of independent form factors. Thus, in this limit for B decays to pseudoscalar (scalar) light mesons three decay form factors can be parametrized by one invariant function $\zeta_{P,S}(q^2)$, and for B decays to vector (axial-vector, tensor) light mesons seven decay form factors can be expressed through two invariant functions $\zeta^{\perp}_{V,A,T}(q^2)$ and $\zeta^{\parallel}_{V,A,T}(q^2)$ for each decay, respectively. This establishes relations between decay form factors at the large recoil of the final light meson, which are obtained with the complete account of second order corrections in the light to B meson mass ratio.

The important consequence of these equations are the well known Isgur-Wise [2] relations (47) between form factors of semileptonic and rare radiative B decays, which were originally obtained for small values of the recoil momentum, and now they are established near the point of maximum recoil if all contributions quadratic in final meson mass are included. The relations (48) obtained by Soares in the constituent quark model [8] also follow from the large recoil symmetry, while the fulfilment of the other ones (49) requires additional relation between the transverse and longitudinal functions (50).

We tested the fulfilment of the large recoil symmetry relations between the B decay form factors in the framework of the relativistic quark model [5,6,15] based on the quasipotential approach in quantum field theory. It was found that they are exactly satisfied in the appropriate limits and corresponding invariant functions were determined. The additional relation (50) is also satisfied in our model. Estimates of the heavy-to-light form factors at $q^2 = 0$ in our model show that their values with the account of $1/m_b$ corrections can differ from the ones in the heavy quark and large recoil limit by $10 - 15\%$. E. g. the form factor $T_1^{K^*}(0)$ responsible for the rare radiative $B \to K^*\gamma$ decay is reduced by $\sim 12\%$ in the large recoil limit. Such reduction of the form factor almost compensates the enhancement of this decay rate by next-to-leading order QCD corrections calculated in the same limit [16,17].

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REFERENCES

[1] A. V. Manohar and M. B. Wise, *Heavy Quark Physics* (Cambridge University Press, Cambridge, 2000).

[2] N. Isgur and M. B. Wise, Phys. Rev. D **42**, 2388 (1990).

[3] M. J. Dugan and B. Grinstein, Phys. Lett. B **255**, 583 (1991).

[4] U. Aglietti, Phys. Lett. B **292**, 424 (1992); U. Aglietti and G. Corbo, Phys. Lett. B **431**, 166 (1998).

[5] R. N. Faustov and V. O. Galkin, Phys. Rev. D **52**, 5131 (1995).

[6] R. N. Faustov, V. O. Galkin and A. Yu. Mishurov, Phys. Rev. D **53**, 6302 (1996).

[7] B. Stech, Phys. Lett. B **354**, 447 (1995).

[8] J. M. Soares, Phys. Rev. D **54**, 6837 (1996).

[9] J. Charles *et al.*, Phys. Rev. D **60**, 014001 (1999).

[10] C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001).

[11] M. Beneke and Th. Feldmann, Nucl. Phys. B **592**, 3 (2001).

[12] J. M. Soares, hep-ph/9810402; hep-ph/9810421 (1998).

[13] G. Burdman and G. Hiller, Phys. Rev. D **63**, 113008 (2001).

[14] G. Burdman and J. F. Donoghue, Phys. Lett. B **270**, 55 (1991).

[15] D. Ebert, R. N. Faustov, V. O. Galkin and H. Toki, hep-ph/0104264 (2001); Phys. Lett. B **495**, 309 (2000).

[16] M. Beneke, T. Feldmann and D. Seidel, hep-ph/0106067 (2001).

[17] S. W. Bosch and G. Buchalla, hep-ph/0106081 (2001).

[18] P. Ball and V. M. Braun, Phys. Rev. D **58**, 094016 (1998).

[19] CLEO Collaboration, T. E. Coan *et al.*, Phys. Rev. Lett. **84**, 5283 (2000).

[20] Belle Collaboration, Y. Ushiroda, hep-ex/014045 (2001).

[21] BABAR Collaboration, M. Convery, hep-ex/0106043 (2001).

[22] A. Ali and A. Ya. Parkhomenko, hep-ph/0105302 (2001).

TABLES

TABLE I. Values of invariant functions ζ_P, $\zeta_V^\perp$ and $\zeta_V^\parallel$ parametrizing B decays to ground state and radially excited pseudoscalar and vector light mesons at the maximum recoil of the final light meson calculated in the relativistic quark model.

Decay	$\zeta_P(E_F^{\rm max})$	Decay	$\zeta_V^\perp(E_F^{\rm max})$	$\zeta_V^\parallel(E_F^{\rm max})$
$B \to \pi$	0.19	$B \to \rho$	0.24	0.25
$B \to K$	0.26	$B \to K^*$	0.28	0.29
$B \to \pi(1430)$	0.24	$B \to \rho(1450)$	0.25	0.27
$B \to K(1460)$	0.19	$B \to K^*(1410)$	0.20	0.21

TABLE II. Values of invariant functions ζ_S, $\zeta_{A,T}^\perp$ and $\zeta_{A,T}^\parallel$ parametrizing B decays to orbitally excited (P-wave) scalar, axial vector and tensor light mesons at the maximum recoil of the final light meson calculated in the relativistic quark model.

Decay	$\zeta_S(E_F^{\rm max})$	Decay	$\zeta_A^\perp(E_F^{\rm max})$	$\zeta_A^\parallel(E_F^{\rm max})$	Decay	$\zeta_T^\perp(E_F^{\rm max})$	$\zeta_T^\parallel(E_F^{\rm max})$
$B \to a_0(1450)$	0.14	$B \to b_1(1235)$	0.08	-0.08			
$B \to K_0^*(1430)$	0.14	$B \to K_1^*(1270)$	0.14	-0.15			
		$B \to a_1(1260)$	0.23	0.49	$B \to a_2(1320)$	0.25	0.27
		$B \to K_1(1400)$	0.20	0.43	$B \to K_2^*(1430)$	0.27	0.29